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An investigation on the S_u–N_{SPT} correlation using GMDH type neural networks and genetic algorithms

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ABSTRACT

The Standard Penetration Test (SPT) is perhaps one of the most effective tests for quick and inexpensive evaluation of the mechanical properties of soil layers. There have been numerous studies directed towards establishment of correction factors for SPT blow count (N_{SPT}) and correlations between N_{SPT} and the properties of cohesionless soils. However, the test method is commonly used in all types of soils. It is, therefore, necessary to investigate the applicability of the correction factors and develop the appropriate correlations for fine-grained soils.

In order to investigate the relevancy of the overburden correction factor for N_{SPT} in fine-grained soils, as well as establishing a correlation between undrained shear strength of such soils with N_{SPT}, a data bank of SPT results on low plasticity fine-grained soils has been compiled. The effect of natural moisture content, plasticity index and effective overburden stress on the correlation of SPT–N_{60} and undrained shear strength of the soils has been studied by the use of Group Method of Data Handling (GMDH) type neural network optimized with genetic algorithms (GA).

Through this study a correlation has been obtained, expressing undrained shear strength of low-plasticity ($PI<20$) fine-grained soils in terms of SPT–N_{60}, PI and effective overburden stress. It has also been shown that natural moisture content has negligible effect on the correlation. The performance of this correlation was compared with other available correlations for this type of soil, and it has been shown that appreciable improvement in prediction of the output has been achieved.

1. Introduction

Evaluation of the mechanical properties of soil layers is the major concern in geotechnical engineering and a basic requirement of any field and laboratory investigation. Various methods and test procedures have been developed for this purpose. However, each category of tests has some drawbacks.

Due to the lack of meticulous theoretical formulization of the correlation between the results of in situ tests and engineering parameters of soil, the only plausible method is empirical derivation based on various regression procedures. Perhaps due to its simplicity Standard Penetration Test (SPT) has received the greatest attention amongst the in situ tests from both academic researchers and professional geotechnical engineers, and is thought to remain as an essential part of soil exploration practice for decades to come (Horn, 1979). However, contrary to the implication by its name, the SPT is not completely standardized (Clayton, 1995; Sivrikaya and Toğrol, 2006) and its results are affected by many factors such as test equipment, drilling procedure, as well as soil types and conditions. This fact has brought about the need for correction of test results.

McGregor and Duncan (1998) have presented the most comprehensive equation for SPT–N correction;

$$N_{60} = (C_B C_C C_R C_{BF} C_S C_A C_E) N_{field} \quad (1)$$

where:

C_B borehole diameter correction factor,
C_C hammer cushion correction factor,
C_R rod length correction factor,
C_{BF} blow count frequency correction factor,
C_S liner correction factor,
C_A anvil correction factor,
C_E energy correction factor.

In addition to the above mentioned correction factors, the effect of overburden pressure can also be accounted for by inclusion of C_N correction factor, adjusting the blow count for 100 kPa effective overburden stress. However, since this correction factor was initially intended for sandy soils, there are differing opinions about the application of this factor to fine-grained soils. It is argued that in fine-grained soils, undrained condition exists during the test, and thus

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correction for the effect of overburden stress is not justified (Saran, 1996; McGregor and Duncan, 1998). Nevertheless, Sivrikaya and Toğrol (2006) have quoted that such correction is useful at deep conditions. Schmertmann (1975) has suggested that an increase in overburden stress might produce proportional increase in undrained shear strength of the soil. Furthermore, it has been suggested previously (Skempton, 1954; Hansbo, 1957) that undrained shear strength of fine-grained soils does depend on effective overburden stress. Now, as the SPT is commonly used in all types of soils, it is of great importance to geotechnical engineers to know whether it is necessary for fine-grained soils to correct the blow counts for effective overburden stress or not.

A different possible approach to this issue is to leave the effective overburden stress out of the above-mentioned correction procedure and instead include any possible effect into the correlation of SPT–N with shear strength parameter. It has also been shown that the plasticity of fine-grained soils influences the correlation of SPT–N with shear strength parameters (be it corrected or not). Adding the possible effect of moisture content onto such correlation produces a multi-parametric problem. To the authors' knowledge, there has not been a study considering the effect of all these parameters simultaneously.

The inter-dependencies of the involved factors in such problems prohibit the use of simple regression analysis and need a more extensive and sophisticated method. A possible research approach to the issue is to use Artificial Neural Networks (ANN). This method seems to be a viable method for development of correlations in a multi-parametric problem. Sonmez et al. (2006) has outlined the advantages of ANN-based models over multiple regression-based models.

System identification techniques are applied in many fields in order to model and predict the behaviors of unknown and/or very complex systems based on given input-output data (Söderström and Stoica, 2002). In this way, soft-computing methods, which concern computation in an imprecise environment, have gained significant attention (Sanchez et al., 1997). Many research efforts have been expended to use evolutionary methods as effective tools of soft-computing methods for system identification such as those by Kristinson and Dumont (1992), Koza (1992), Iba et al. (1993), and Rodríguez-Vázquez (1999). Among these methodologies, Group Method of Data Handling (GMDH) algorithm is a self-organizing approach by which gradually complicated models are generated based on the evaluation of their performances on a set of multi-input-single-output data pairs (x_i, y_i) $(i=1, 2, \ldots, M)$. The GMDH was first developed by Ivakhnenko (1971) as a multivariate analysis method for complex system modelling and identification, which can be used to model complex systems without having specific knowledge of the systems. The main idea of GMDH is to build an analytical function in a feedforward network based on a quadratic node transfer function whose coefficients are obtained using regression technique (Farlow, 1984). In recent years, however, the use of such self-organizing networks has led to successful application of the GMDH-type algorithm in a broad range of areas in engineering, science, and economics (e.g. Farlow, 1984; Iba et al., 1996; Mueller and Lemke, 2000; Nariman-Zadeh et al., 2002, 2003 and 2005).

Moreover, there have been many efforts in recent years to deploy genetic algorithms in the design of artificial neural networks (Porto, 1997; Yao, 1999). Recently, genetic algorithms have been used in a feedforward GMDH-type neural network for each neuron searching its optimal set of connection with the preceding layer (Vasechkina and Yarin, 2001; Nariman-Zadeh et al., 2003). Over the last few years, the ANN has been applied to many geotechnical engineering problems (Zhu et al., 1998; Goh, 1995; Yang and Rosenbaum, 2002; Lee, 2003; Najjar and Basheer et al., 1996) and has demonstrated some degree of success (Shahin et al., 2001).

In this paper, GMDH type neural networks optimized using genetic algorithms (GAs) are used to model the effects of plasticity index, natural moisture content, effective overburden stress, and SPT–N_{60} value as input parameters on undrained shear strength of low plasticity clays using 80 sets of experimentally obtained training and test data. Sensitivity analysis of the obtained model has been carried out to study the influence of input parameters on model output. Thereof, a chart has been produced to estimate undrained shear strength of low plasticity clays using obtained results from the sensitivity analysis. Finally, the results of proposed correlation are compared with that of other correlations.

2. Review of previously proposed correlations

Consistency and strength of fine-grained soils is usually determined by either unconfined compression (UC) test or unconsolidated-undrained (UU) triaxial test. The result of unconfined compression test is expressed as undrained compressive strength (q_u) and due to its simplicity is very popular amongst geotechnical engineers. For this reason, many of the researchers have proposed correlations between q_u and SPT–N value. The earliest q_u–N_{SPT} correlation is given by Terzaghi and Peck (1967). They produced the relation given in Table 1, which in effect means $q_u = 12.5 \times N_{SPT}$. This appears to be a mean value for various types of fine-grained soils and does not take any other factors (e.g. PI) into consideration.

The first attempts at including the effect of index properties of fine-grained soils in the q_u–N_{SPT} relation can be traced back to early 1970's when Sanglerat (1972) suggested different relationships between q_u and N_{SPT} for clays and silty clays. Based on previous works, Schmertmann (1975) produced a correlation chart from which it can be concluded that for a constant SPT–N, q_u increases with PI. Sivrikaya and Toğrol (2006) have also proposed a PI dependent correlation for q_u–N_{field} as well as q_u–N_{60}. A summary of the existing correlations are presented in Table 2. Besides the correlations expressing q_u in terms of N_{SPT}, a number of other researchers considered the correlation between undrained shear strength (S_u) with N_{SPT}. It is worth mentioning that by assuming full saturation of the sample in unconfined compression test the failure envelope may be taken to be parallel to σ_n axis (i.e. $\phi_u = 0$) and thus undrained cohesion or shear strength is equal to $q_u/2$.

One of the first attempts at expressing S_u (obtained from UU compression tests) in terms of SPT–N value dates back to 1974 when Stroud proposed three slightly different coefficients for low, medium and high plasticity clays. Contrary to the other works, Stroud suggested that for a constant SPT–N value, S_u decreases with increase in PI. On a slightly different approach, Décourt (1990) proposed other correlations between S_u and N_{field} and N_{60}. In a comprehensive study by Sivrikaya and Toğrol (2006), a number of S_u–N_{field} and S_u–N_{60} correlations based on the results of unconsolidated-undrained (UU) triaxial tests, were proposed for various types of fine-grained soils. A summary of the proposed correlations are presented in Table 3.

3. Modelling using GMDH type neural networks

By means of GMDH algorithm a model can be represented as a set of neurons in which different pairs in each layer are connected through a quadratic polynomial and thus produce new neurons in the

Table 1
q_u–N_{SPT} relations for fine-grained soils in accordance with consistency (Terzaghi and Peck, 1967)

Consistency	N_{SPT}	q_u (kPa)
Very soft	<2	<25
Soft	2–4	25–50
Medium	4–8	50–100
Stiff	8–15	100–200
Very stiff	15–30	200–400
Hard	>30	>400

Table 2
Correlations between q_u and SPT–N for various fine-grained soils

Author(s)	Soil type	q_u (kPa)
Terzaghi and Peck(1967)	Fine-grained soil	$12.5N$
Sanglerat (1972)	Clay	$25N$
	Silty clay	$20N$
Schmertmann (1975)	High *PI* clays	$25N$
	Medium *PI* clay	$15N$
	Low *PI* clay	$7.5N$
Nixon (1982)	Clay	$24N$
Kulhawy and Mayne (1990)	Fine-grained soil	$58N^{0.72}$
Sivrikaya and Toğrol (2006)	Highly plastic clay	$9.7N_{\text{field}}$
		$13.63N_{60}$
	Low plastic clay	$6.7N_{\text{field}}$
		$9.86N_{60}$
	Clay	$8.66N_{\text{field}}$
		$12.38N_{60}$
	Fine-grained soil	$8.64N_{\text{field}}$
		$12.36N_{60}$

next layer. Such representation can be used to map inputs to outputs. The formal definition of the identification problem is to find a function $\hat{f}$ that can be approximately used instead of the actual one, f in order to predict output $\hat{y}$ for a given input vector $X=(x_1, x_2, x_3, \ldots, x_n)$ as close as possible to its actual output y. Therefore, given M observation of multi-input-single-output data pairs:

$$y_i = f(x_{i1}, x_{i2}, x_{i3}, \ldots, x_{in})(i = 1, 2 \ldots M). \tag{2}$$

It is now possible to train a GMDH type neural network to predict the output values $\hat{y}$ for any given input vector $X=(x_{il}, x_{i2}, x_{i3}, \ldots, x_{in})$, that is:

$$\hat{y}_i = \hat{f}(x_{i1}, x_{i2}, x_{i3} \ldots, x_{in})(i = 1, 2 \ldots M). \tag{3}$$

The problem is now to determine a GMDH type neural network so that the square of difference between the actual output and the predicted one is minimised, that is:

$$\sum_{i=1}^{M} \left[\hat{f}(x_{i1}, x_{i2}, x_{i3}, \ldots x_{in}) - y_i\right]^2 \rightarrow \min. \tag{4}$$

General connection between inputs and output variables can be expressed by a complicated discrete form of the Volterra functional series in the form of:

$$y = a_0 + \sum_{i=1}^{n} a_i x_i + \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} x_i x_j + \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} a_{ijk} x_i x_j x_k + \ldots, \tag{5}$$

which is known as the Kolmogorov–Gabor polynomial (Ivakhnenko, 1971; Farlow, 1984; Iba et al., 1996; Sanchez et al., 1997; Nariman-Zadeh et al., 2003). This full form of mathematical description can be represented by a system of partial quadratic polynomials consisting of only two variables (neurons) in the form of:

$$\hat{y} = G(x_i, x_j) = a_0 + a_1 x_i + a_2 x_j + a_3 x_i^2 + a_4 x_j^2 + a_5 x_i x_j. \tag{6}$$

There are two main concepts involved within GMDH type neural networks design, namely, the parametric and the structural identification problems. In this way, some works by Nariman-Zadeh, et al., (2002, 2003 and 2005) present hybrid GA and singular value decomposition (SVD) method to optimally design such polynomial neural networks. The methodology in these references has been successfully used in this paper.

A non-commercial code (GEvoM) for the evolved GMDH type neural network has been developed by the third author and details about the code and general description of the technique may be found in the following web site: http://research.guilan.ac.ir/gevom.

4. The database

The data used in this study were gathered from the National Iranian Geotechnical Database, which has been set up in the Building and Housing Research Centre (BHRC) (Kalantary, 2005).The database has been established under a mandate from the Management and Planning Organization (MPORG), which supervises the professional activities of all of the consultancy firms in Iran.

The data compiled in the database have been extracted from routine geotechnical investigation reports by accredited geotechnical engineering consultancy firms. It is common practice by these firms to use either rotary or percussion drilling methods. However, for the purposes of this study, meticulous care was taken to exclude any data from doubtful procedure and/or non-standard drilling and sampling methods.

Furthermore, as a precautionary measure, only the results obtained from rotary drilling procedure were selected. This selection criterion has a twofold benefit. Firstly, since the borehole size in this type of drilling is usually less than 116 mm, no correction factor is needed for borehole size (i.e. $C_B=1$), and secondly, since Shelby tube undisturbed sampler is commonly used in rotary drilling, it can be ensured that the data used in this study originated from a uniform procedure. It must also be mentioned that as a general practice, safety hammers are used by most of the consultancy firms in Iran and the SPT sampler is not usually fitted with liner (i.e. $C_S=1$). Hence, by neglecting C_C, C_{BF} and C_A, only the correction factors relating to rod length and energy correction factor of the hammer were considered in this study. Thus:

$$N_{60} = (0.83 \times C_R) N_{\text{field}}. \tag{7}$$

The extracted data from the reports are kept in a standardized format in the databank. Under this format, various data strings may be simultaneously specified for data search. For the purposes of the present study, a number of criteria were set for data selection. Initially, all of the available SPT results in fine-grained soils with the classification of CL, CH, ML, MH and CL-ML were identified. From amongst these data, those that had the other relevant data to this study were virtually separated. The relevant data included plasticity index (*PI*), natural moisture content (W_n) and undrained shear strength evaluated from UU triaxial test in the same layer. In addition to the above mentioned parameters, the unit weight of the overlaying layers must also be known.

A total of 436 cases which conformed to the basic selection criteria were gathered. Most of these fell within the low plastic clay category. Due to the fact that undisturbed sampling of this type of soil is difficult (Peters, 1988) and it was expected that some of the triaxial test results might give misleading information about the shear strength of the samples, a number of measures were deemed necessary to

Table 3
Relations between SPT–N and S_u for fine-grained soils

Author	Soil type	S_u (kPa)
Stroud (1974)	$PI<20$	$6-7\ N$
	$20<PI<30$	$4-5\ N$
	$PI>30$	$\approx 4.2\ N$
Décourt (1990)	Clay	$12.5\ N$
	Clay	$15\ N_{60}$
Sivrikaya and Toğrol (2006)	CL	$3.97\ N_{\text{field}}$
		$5.82\ N_{60}$
	CH	$5.9\ N_{\text{field}}$
		$8.76\ N_{60}$
	Clay	$5.13\ N_{\text{field}}$
		$7.57\ N_{60}$
	Fine grained soil	$4.68\ N_{\text{field}}$
		$6.97\ N_{60}$

Fig. 1. Geographical distribution of the data on the map of Iran.

overcome this issue. It was decided to exclude all datasets with $N<5$, since weak samples were more prone to disturbance. Furthermore, all samples that did not have clear descriptions and/or photographs were excluded. As a result of the data filtering process, the total number of the remaining cases was reduced to 80. These data related to samples that had no visible fissures or deformities.

It is worth mentioning that the datasets finally used in this study were extracted from 39 different geotechnical investigation reports produced by eight well-know consultancy firms in the past 10 years. Geographical distribution of the data is shown on the map of Iran in Fig. 1. Distribution of SPT–N_{60}, C_u, PI, and W_n with depth is presented in Fig. 2. Furthermore, in order to provide a general view on the degree

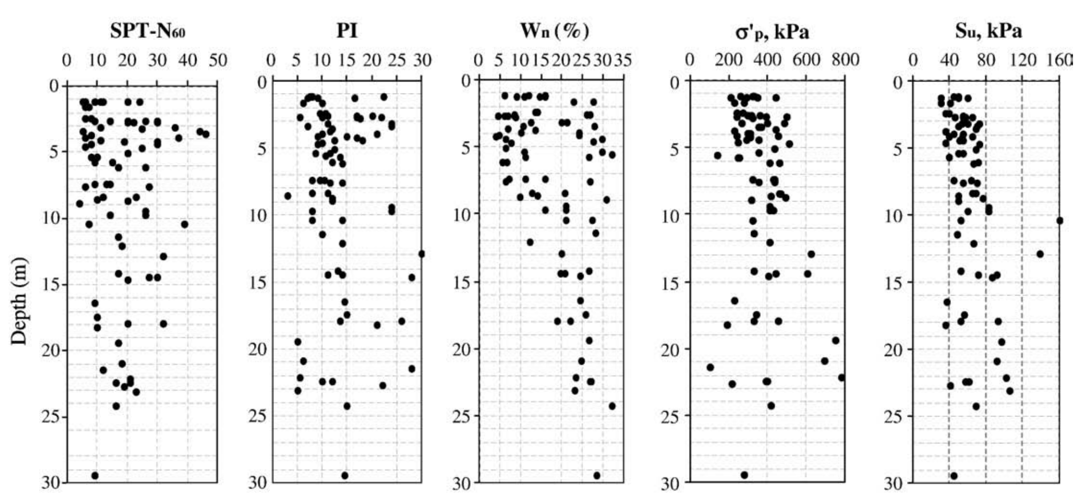

Fig. 2. Distribution of SPT–N_{60}, PI, W_n, σ'_p and S_u, with depth.

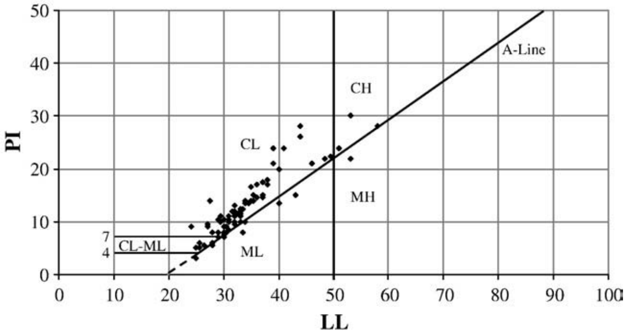

Fig. 3. Distribution of plasticity characteristics of the samples.

of consolidation of the samples, the pre-consolidation pressures of the samples were evaluated by using the following empirical correlation (NAVFAC DM-7.1, 1982):

$$\sigma'_p = \frac{S_u}{0.11 + 0.0037\mathrm{PI}}. \tag{8}$$

The variation of pre-consolidation pressure with depth is also shown in Fig. 2. Most of the SPT–N_{60} and S_u lay between 5 and 30, and 35–100 kPa range respectively. It is also worth noting that most of the data are from tests on low-plasticity clays (CL). Distribution of plasticity characteristics of the samples are shown in Fig. 3.

5. Modelling of S_u by GMDH type neural networks

Modelling of the undrained shear strength of fine-grained soils (S_u) by GMDH type neural networks require identification of appropriate input parameters. It has been shown by many researchers (Stroud, 1974; Schmertmann, 1975; Sivrikaya and Toğrol, 2006) that the plasticity index (*PI*) influences the correlation between undrained shear strength of the fine-grained soils and corrected SPT blow counts (N_{60}). Furthermore, as mentioned in the Introduction, it is likely that the effective overburden stress (σ'_n) also affects the SPT–*N* in this type of soils, and thus it ought to be included in the investigation. The natural moisture content (W_n) also influences the in situ shear strength of fine-grained soils and thereby it could affect the SPT–*N*.

In view of the fact that the evolutionary process of selecting the configuration of GMDH type networks selects the best possible combination of inputs, it does not necessarily use all the inputs in constructing the model. In other words, the GMDH type networks is intrinsically capable of determining the degree of influence of each input parameters on the model output and thus any possible redundancy of each parameter will automatically be determined. Thus, the above-mentioned variables (*PI*, N_{60}, σ'_n and W_n) were chosen as the primary input parameters.

As mentioned in the previous section, 80 cases obtained from the experimental data (input-output pairs) were selected. However, in order to demonstrate the prediction ability of the evolved GMDH type neural networks, the data have been divided into two different sets, namely, training and testing sets. The training set, which consists of 60 out of 80 inputs-output data pairs, is used for training the neural network models using the evolutionary method of this paper. The testing set, which consists of 20 unforeseen input-output data samples during the training process, is merely used for testing to show the prediction ability of such evolved GMDH type neural network models during the training process.

The GMDH type neural networks are now used for such input-output data to find the polynomial model of undrained shear strength of low plasticity clays in respect to its effective input parameters. In order to genetically design such GMDH type neural network described in the previous section a population of 50 individuals with a crossover probability of 0.7 and mutation probability of 0.07 has been used in 300 generations that no further improvement has been achieved for such population size. The structure of the evolved 3-hidden layer GMDH type neural networks is shown in Fig. 4 corresponding to the genome representations of *ababacccbcadbdcd* for undrained shear strength of low plasticity clays in which *a*, *b*, *c*, and *d* stand for plasticity index (*PI*), natural moisture content (W_n), effective overburden stress (σ'_n), and SPT–N_{60} value, respectively. The good behavior of the GMDH type neural network model is also depicted in Fig. 5 for testing data of undrained shear strength of low plasticity clays.

It is clearly evident that the evolved GMDH type neural network in terms of simple polynomial equations could successfully model and predict the output of testing data that has not been used during the training process. As a measure of the accuracy of the model, root mean squared errors (RMSE) of both training and testing sets of data are evaluated to be 7.4 and 8.8 kPa, respectively.

6. Sensitivity analysis

The polynomial model produced by the evolved GMDH type neural network is in the form of a complex equation and thus the effect of each parameter cannot directly be examined. Instead, the sensitivity of the model to each parameter is evaluated by examining the variation of one parameter with respect to the specified parameter with constant values for the remaining variables. Since extrapolation may result in an erroneous outcome, the variation of each parameter is kept within the bounds of the input data range.

In other words the considered variation of N_{60}, ,σ'_n *PI*, and W_n is limited to the values stated below:

N_{60} = 5–30,
σ'_n = 50–250 kPa,
PI = 5–20%,
W_n = 5–25%.

Limiting $N_{60} \geq 5$, meant that the equation of the correlation did not necessarily pass through the origin (i.e. $S_u = 0$). Obviously, this condition could be imposed in the modelling. However, it was noted that this had an adverse effect on the overall performance of the model. Hence, the sensitivity analysis was carried out within the bounds of the above-mentioned data limits.

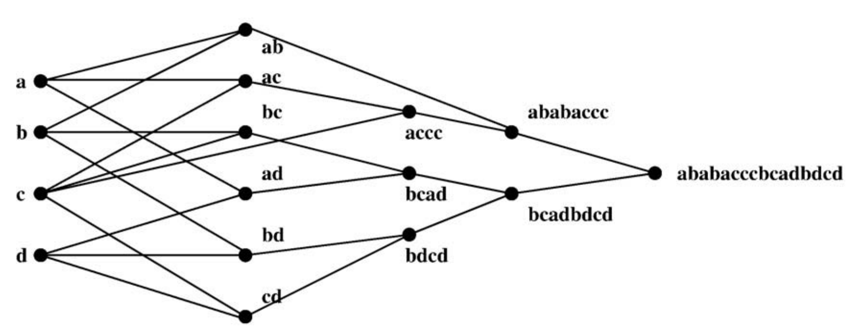

Fig. 4. Evolved structure of the generalized GMDH neural network for undrained shear strength of low plasticity clays.

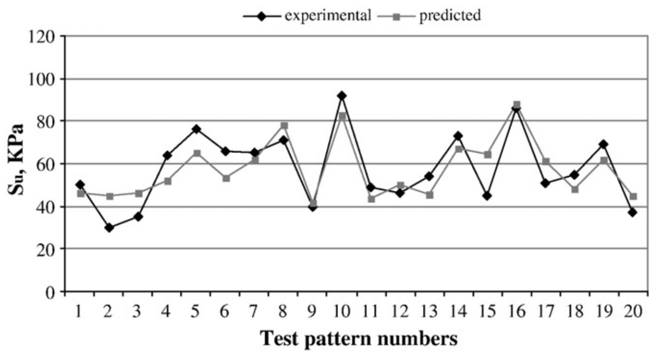

Fig. 5. Comparison of experimental values of S_u of low plasticity clays with the predicted values using evolved GMDH neural networks for testing data.

First of all the effect of moisture content on the variation of S_u–N_{60} was examined and it was noticed that this parameter does not greatly influence the correlation. Fig. 6 shows the variation of S_u–N_{60} under the 100 kPa effective overburden stress for various *PI* values, with the upper and lower values of moisture content. It can be noted that the trend of variation with high and low moisture content is the same and the maximum difference occurs at low values of SPT–N_{60}. Since the maximum difference at N_{60} = 5 is about 15 kPa, it can be concluded that moisture content has little bearing on the S_u–N_{60} correlation.

The second parameter considered was effective overburden stress. Variations of S_u–N_{60} under different overburden stresses are presented in Fig. 7. In these figures the moisture content was 10%. It can clearly be noticed that at low values of N_{60}, overburden stress has no effect on the S_u–N_{60} correlation, whereas at high values of N_{60}, marked differences can be seen. In other words, for $N_{60}>15$, overburden stress greatly affected the correlation of S_u–N_{60} in as much as S_u can increase by three times.

Finally, the effect of plasticity index on correlation of S_u–N_{60} is examined. Fig. 8 a and b show the variations of S_u against *PI* for different N_{60} values under 100 and 250 kPa effective overburden stress, respectively. It can be noticed from these figures that at very low values of *PI*, the S_u–N_{60} correlation is greatly affected by *PI*, whereas at $15<PI<20$ the correlation is almost not affected by *PI*. Inversely, at high values of N_{60} ($N_{60}>15$), *PI* has a major effect on the correlation. It can also be concluded from these two figures that S_u increases with N_{60} at a decreasing rate with respect to increase of *PI*. Furthermore, for a constant N_{60} value greater than 15 the estimated values of undrained shear strength decreases with increase in *PI* whereas for low values of N_{60}, undrained shear strength increases slightly with *PI* value up to *PI* = 10. In all of the figures (Figs. 6–8), the natural trend of increase for S_u with N_{60} exists.

7. Proposed method

In view of the insight gained by the sensitivity analysis, it is proposed to express N_{60} in terms of the mean values of undrained shear strength S_u for various moisture contents, since it was determined

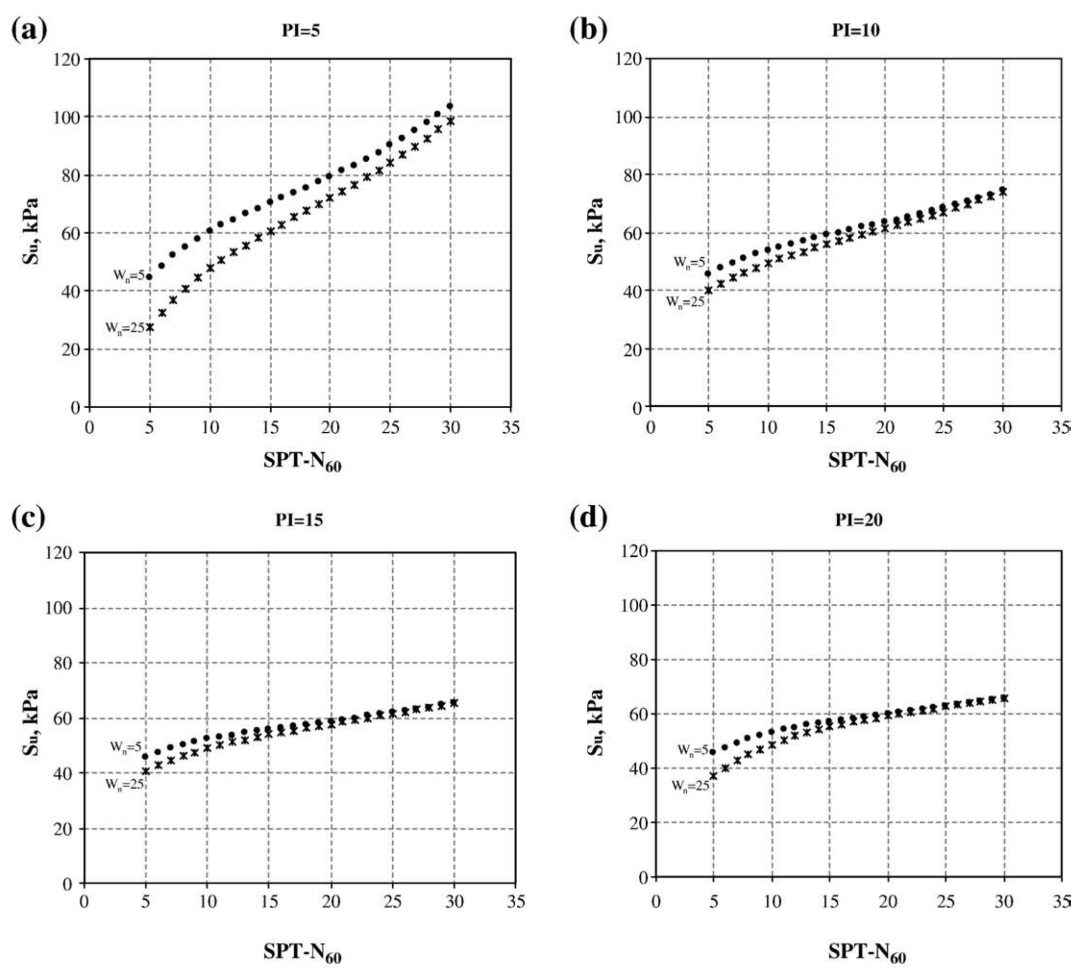

Fig. 6. Variation of S_u–N_{60} under 100 kPa overburden stress for various *PI* values, with the upper and lower values of moisture content.

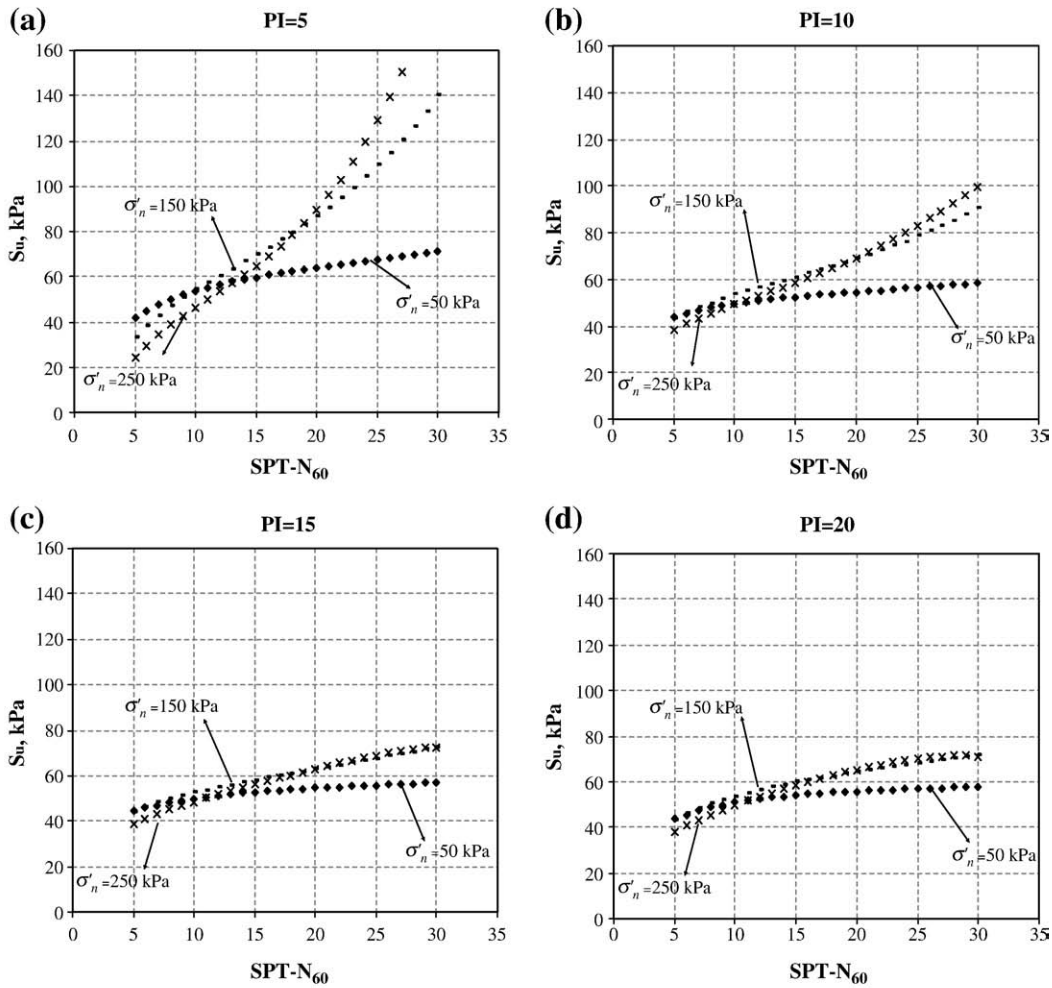

Fig. 7. Variation of S_u–N_{60} under different effective overburden stress.

that moisture content has negligible effect on this variation. By inspection, it can be noted that a relatively good fit may be obtained by linear regression. This is repeated for every *PI* (5, 10, 15 and 20) and σ'_n (50, 100, 150, 200 and 250 kPa) value and thus 20 linear equations may be obtained. The coefficients of determination (R^2) for the fitted lines are all above 0.8 and for half of them; the R^2 value was above 0.95. These values indicate relatively good fit. The coefficients of these equations have subsequently been evaluated and their variation with *PI* values is determined. It was found that a second order polynomial produces the best fit to this variation with an average R^2=0.97. Thereby, five equations for the five effective overburden stresses were obtained, expressing the S_u in terms of N_{60} and *PI* value:

$$\sigma'_N = 50\text{kPa} \qquad S_u = \mathrm{PI}^2(0.0055N - 0.008) - \mathrm{PI}(0.176N - 0.3) + 1.7N + 41 \quad (9-\mathrm{a})$$

$$\sigma'_n = 100\text{kPa} \qquad S_u = \mathrm{PI}^2(0.0125N - 0.1) - \mathrm{PI}(0.425N - 3.4) + 4.1N + 16.5 \quad (9-\mathrm{b})$$

$$\sigma'_n = 150\text{kPa} \qquad S_u = \mathrm{PI}^2(0.0216N - 0.222) - \mathrm{PI}(0.721N - 7.4) + 6.9N - 19 \quad (9-\mathrm{c})$$

$$\sigma'_n = 200\text{kPa} \qquad S_u = \mathrm{PI}^2(0.0306N - 0.33) - \mathrm{PI}(1.016N - 11.1) + 9.4N - 50.5 \quad (9-\mathrm{d})$$

$$\sigma'_n = 250\text{kPa} \qquad S_u = \mathrm{PI}^2(0.0371N - 0.423) - \mathrm{PI}(1.22N - 14) + 11.1N - 75. \quad (9-\mathrm{e})$$

These correlations are plotted in Fig. 9 a–e. From these figures, it may be noted that for N_{60}≈8–12, the correlation is almost independent of *PI* variation. For a constant value of $N_{60}>12$ undrained shear strength appears to decrease with increase of *PI*, whereas for $N_{60}<8$ the reverse is true. Further investigation into the rate of this variation indicated that if undrained shear strength is normalized with respect to the square root of effective overburden stress $\left(S_u/\sqrt{\sigma'_n}\right)$, a unified chart (Fig. 10) for the correlation of S_u–N_{60} may be obtained for $5<PI<20$. It must, however, be emphasized that the validity of this type of correlations outside the considered range needs further verification and will almost certainly follow a different pattern.

8. Comparison

In order to examine the performance of the proposed model, statistical comparison of the response of the correlations proposed by Stroud (1974) and Sivrikaya and Toğrol (2006) and that of the model presented in this study to the available datasets is made.

Out of the 80 cases initially used for the development of the model in this study, 56 cases had CL classification with overburden stress less than 250 kPa. Thus, the N_{field} and N_{60} values of this group of cases were used as input for the appropriate correlations from the above mentioned references, and the undrained shear strength (S_u) estimations were obtained. The predictions of each correlation were compared with the measured values.

Fig. 11 presents this method of comparison. The mean values and the standard deviations (*SD*) are quoted for quantitative assessment. Obviously, the less the scatter around the 1:1 line is, the better the performance of the correlation is. Furthermore, in order to provide

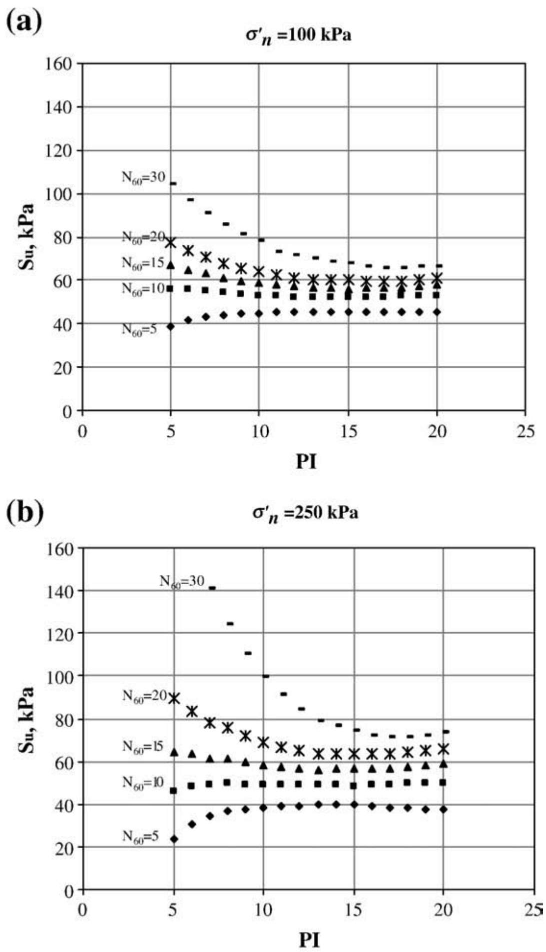

Fig. 8. variation of S_u–PI for different N_{60} values under 100 and 250 kPa effective overburden stress.

better means of visual judgment, two other (broken) lines indicating ± 20% deviation from the perfect agreement are also drawn.

Fig. 11 a and b present the predictions of the correlation proposed by Stroud (1974), using N_{field} and N_{60} respectively. It can be noted that a smaller scatter is encountered if N_{60} is used in the correlation. The mean value and the standard deviation reduce from 1.87 and 0.7 respectively to 1.41 and 0.56. However, both figures indicate that the correlation proposed by Stroud (1974) overestimates the values of S_u.

Fig. 11 c and d present the predictions of the two sets of correlations proposed by Sivrikaya and Toğrol (2006) for N_{field} and N_{60}. It is very interesting to note that not much difference exists between the scatter of the results in the two figures. The mean values of the estimated to measured undrained shear strengths are 1.13 and 1.23, and the standard deviations are 0.48 and 0.54 respectively. This is a clear indication that the two proposed correlations are well adjusted.

Fig. 11 e shows the predication ability of the proposed model. The mean values of both of the above-mentioned correlations are above unity. This indicates the tendency of these correlations to overestimate S_u, whereas, the mean value of the predicted to measured ratio of the S_u by the proposed method in this study is slightly below unity. Furthermore, a marked decrease in the scatter of the results may also be noted (SD=0.25) in comparison with the other two correlations, which can be attributed to enhanced performance due to the inclusion of the extra parameter (namely effective overburden stress) in the modelling.

Moreover, root mean square error (RMSE) and values account for (VAF) of the predicated values by the above-mentioned correlations were calculated and presented in Table 4. The results of these statistical evaluations also confirm the improved accuracy gained by the proposed method.

An alternative method of comparison is to present a log-normal plot of the ratio of the estimated values of S_u to the measured values versus their cumulative average or otherwise known as cumulative probability. Long and Shimel (1989) and Alsamman (1995) have shown that using such statistical presentation will provide valuable insight and quantified measure of the prediction ability of empirical correlations.

Hence, for the current set of data, the ratio of calculated to measured values of undrained shear strength, ($S_{u(\mathrm{e})}/S_{u(\mathrm{m})}$), is arranged in ascending order numbered (1, 2, 3,…i,…n) and a cumulative probability, P, is determined for each undrained shear strength value as:

$$P = \frac{i}{(n+1)} \tag{10}$$

where i is the number of value considered in P. The following points are note-worthy in assessing the bias and dispersion associated with a particular predictive method:

(1) The ratio of calculated to measured value at P=50% probability is a measure of the tendency to overestimate or underestimate the undrained shear strength. The closer the ratio is to unity, the better the agreement.
2) Log-normally distributed data will plot on a straight line
3) The slope of the line through the data points is a measure of the dispersion or standard deviation. The flatter the line is, the better general agreement becomes.

Fig. 12 shows the plot of estimated to measured undrained shear strength, ($S_{u(\mathrm{e})}/S_{u(\mathrm{m})}$), values versus cumulative probability for the present cases. For the probability of 50%, the ($S_{u(\mathrm{e})}/S_{u(\mathrm{m})}$) value for proposed and Sivrikaya and Toğrol (2006) correlations is close to unity, whereas the ratio for Stroud correlation based on N_{field} and N_{60} is about 1.3 and 1.6 respectively, demonstrating a trend toward overestimation.

It can be noted that ($S_{u(\mathrm{e})}/S_{u(\mathrm{m})}$) value at P=50% for the N_{field} based correlation of Sivrikaya is slightly lower than those of N_{60} based correlation. This discrepancy could be as a result of the fact that the SPT results used in this study were obtained by safety hammers, whereas the proposed correlation by Sivrikaya and Toğrol (2006) for S_u–N_{field} were based on the SPT results using donut hammer which has a smaller energy ratio (ERr=0.45) than safety hammer (ERr=0.50) (Clayton, 1990).

Both of the correlations proposed by Stroud and Sivrikaya exhibit a higher dispersion than the proposed correlation, as indicated by the flatter slope of the line. It is obvious that the results for the proposed correlation are closer to log-normal distribution than those for Stroud (1974) and Sivrikaya and Toğrol (2006) correlations.

9. Summary and conclusions

It has been attempted in this study to deploy a powerful system identification technique to develop the S_u–N_{60} correlation. An optimized GMDH type neural network with genetic algorithm has been used to develop a mathematical model that intricately defines the interdependencies of the involved variables.

The sensitivity analysis of the obtained model has been carried out to study the influence of input parameters on model output and

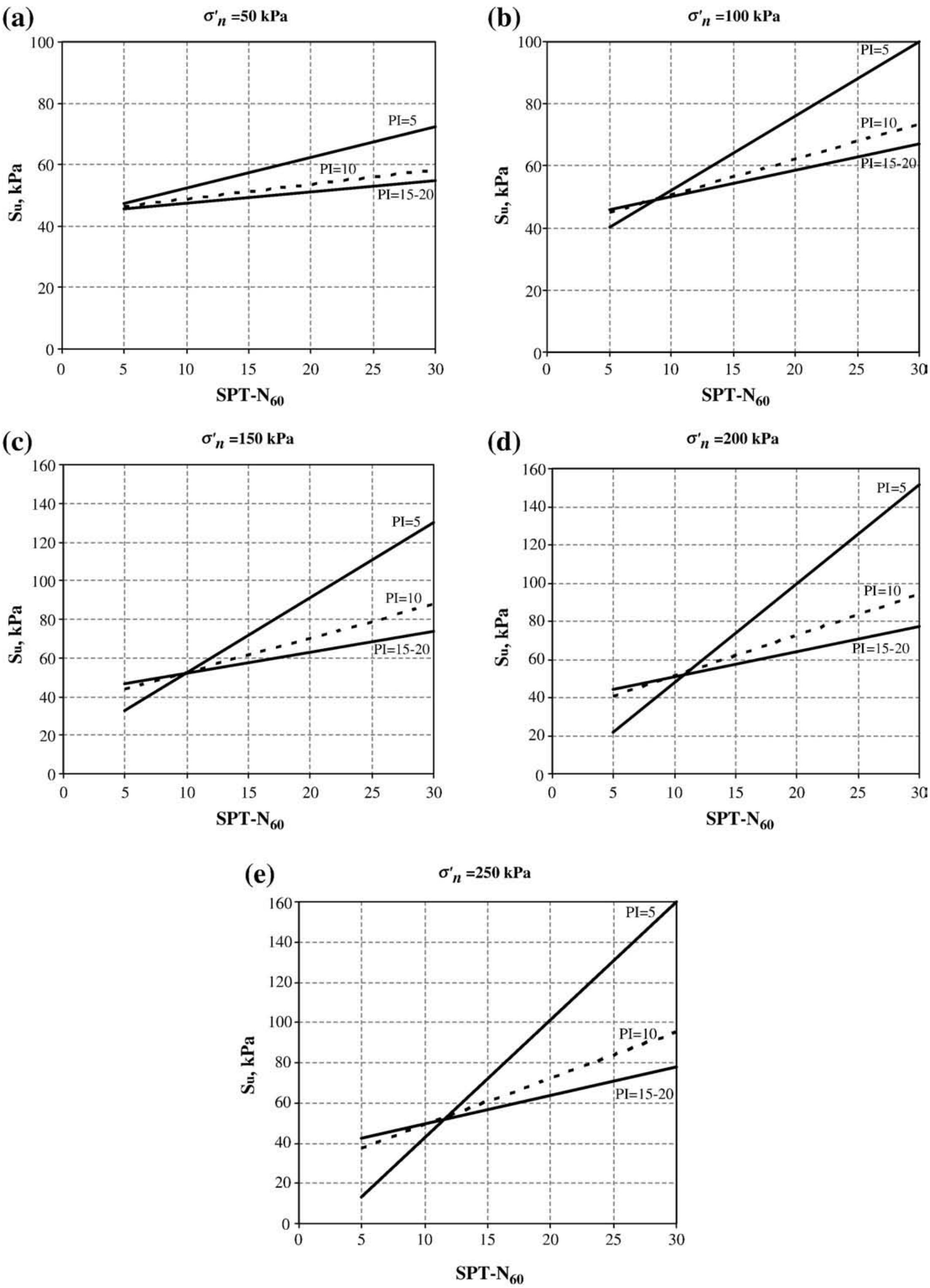

Fig. 9. Proposed S_u–N_{60} correlation for 50, 100, 150, 200 and 250 kPa effective overburden stress.

was limited to the actual bounds of the data range (i.e. N_{60} = 5–30, σ'_n = 50–250 kPa, PI = 5–20%, W_n = 5–25%) in order to minimize the deviation. It is acknowledged that should a wider range of parameters be considered, a varied function would probably be obtained, but within the bounds of the data range, a relatively accurate model has been achieved.

The sensitivity analysis shows that natural moisture content has negligible effect on the correlation, whereas both PI and σ'_n influence the function. The influence of effective overburden stress is the greatest at higher values of SPT–N and lower values of PI.

Considering the knowledge obtained from the sensitivity analysis, a unified chart has been developed. By this chart, undrained shear strength of the soil (S_u) may be determined, provided N_{60}, PI and σ'_n are known. An interesting feature of this chart is that for $N_{60} > 10$, S_u increases with the decrease of PI, whereas for $N_{60} < 10$ the reverse is true. This is in partial agreement with the works of Stroud (1974)

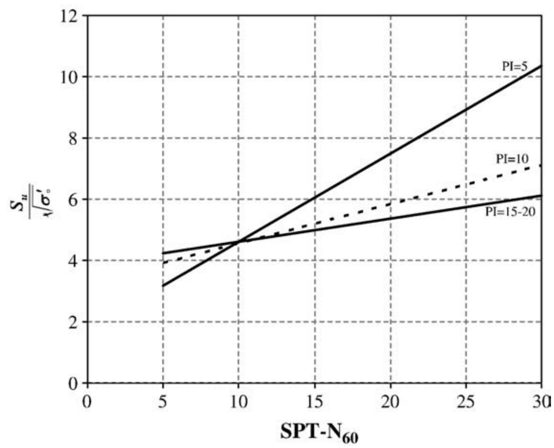

Fig. 10. Proposed correlation between $(S_u/\sqrt{\sigma'_n})$ for low plasticity clays.

who had shown that for a constant value of SPT–*N*, S_u is inversely proportional to *PI*. However, Décourt (1990) and Sivrikaya and Toğrol (2006) have suggested that the opposite is true (i.e. S_u is directly proportional to *PI*).

Finally, the performance of the suggested approach was validated by comparison with the predictions of the correlations proposed by Stroud (1974) and Sivrikaya and Toğrol (2006) for low plasticity clays. The bias and dispersion of the outcomes were presented in a format to facilitate quantified assessment of the performances.

It has been shown that the correlation proposed by Stroud (1974) excessively overestimates the results. However, should N_{60} be used instead of N_{field}, the outcome improves considerably, confirming the notion suggested by McGregor and Duncan (1998), that it is sound to use N_{60} instead of N_{field} for correlations proposed before 1990.

The performance of the correlations proposed by Sivrikaya and Toğrol (2006) for low plasticity clays attained a high rating in this comparison. The predictions of the two correlations based on N_{field} and N_{60} compared well with each other, which indicates that the two correlations are well adjusted.

However, the predictions of the proposed approach show a definite improvement even in comparison with the works of Sivrikaya and Toğrol (2006), which is a clear indication of the merits of the approach.

The data used in this study was mostly limited to low plasticity (CL) clays, hence the use of the obtained model must be restricted to this type of soils. However, the same approach may be applied to an extended database with wider range of input parameters to obtain more comprehensive correlation.

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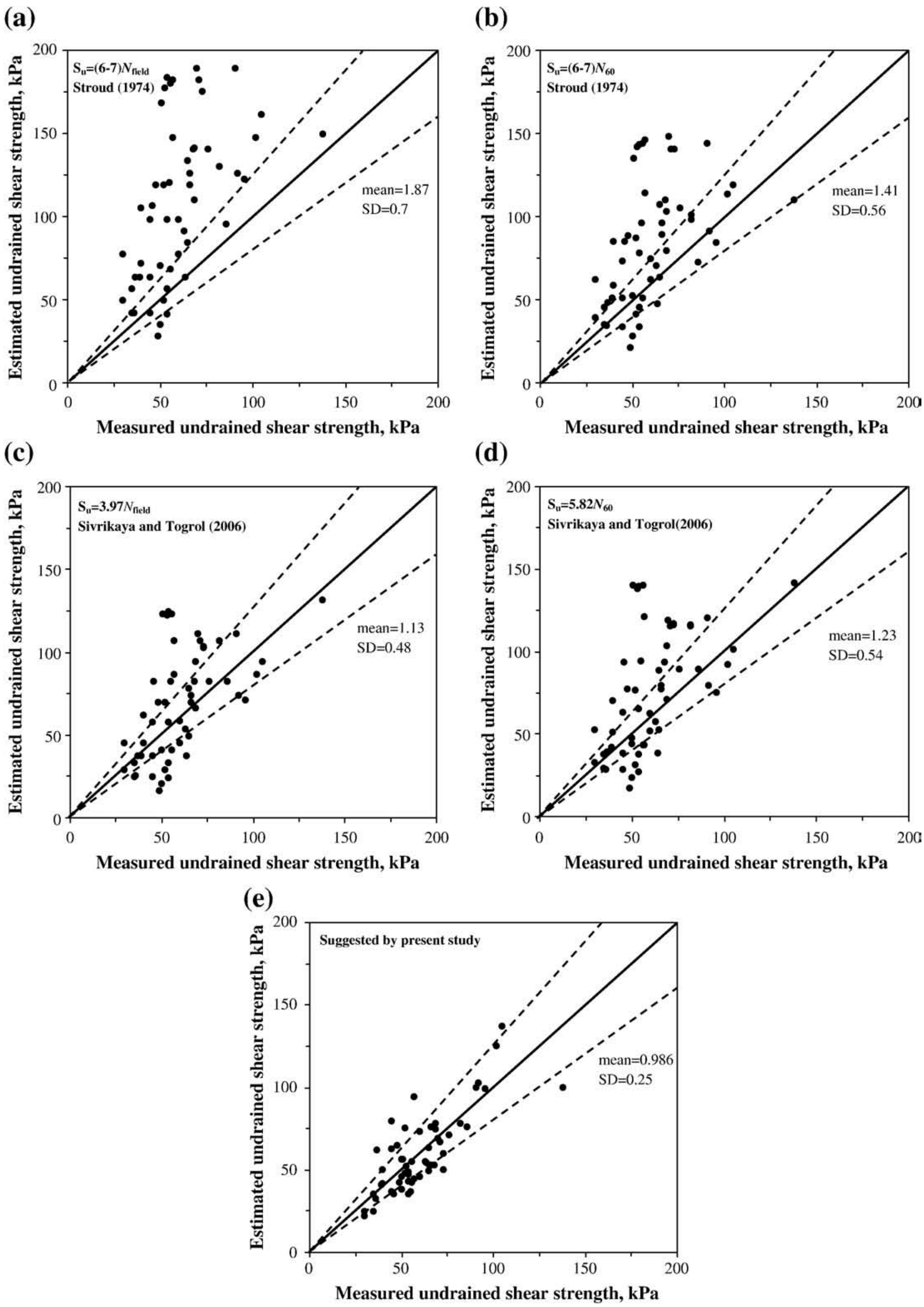

Fig. 11. Measured versus estimated undrained shear strength by different correlations.

Table 4
RMSE and VAF for different correlations

	$S_u=(6–7)N_{field}$ Stroud (1974)	$S_u=(6–7)N_{60}$ Stroud (1974)	$S_u=3.97N_{field}$ Sivrikaya and Toğrol (2006)	$S_u=5.82N_{60}$ Sivrikaya and Toğrol (2006)	Proposed method
RMSE	62.3	39.5	27.2	32.8	14.59
VAF (%)	9	22	38	28	72

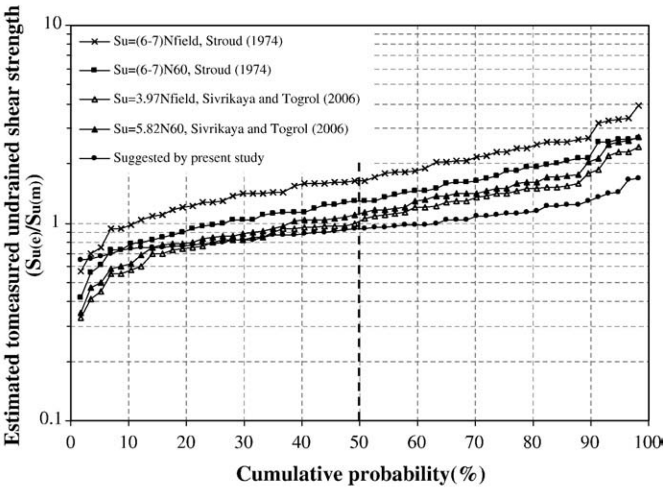

Fig. 12. Comparison of correlations' results using the probability approach.